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Multi-parameter deformed fermionic oscillators

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Abstract. In this paper, we study a quantum group covariant deformed fermion algebra. This system can be formulated in n dimensions and posesses two deformation parameters. The undeformed fermion algebra is obtained when both deformation parameters are unity. When both parameters are zero the deformed fermionic oscillator algebra reduces to the orthofermion algebra. If the quantum group symmetry is not preserved, then the number of parameters in n dimensions can be increased to $2n-2$.

1 Introduction

In recent years, quantum groups, which are q-deformations of Lie groups and Lie algebras [1–4], have found wide interest among mathematicians and physicists. Many studies of q-deformed objects [5–12] have been performed. Deformations of the boson algebra [13–16] played an important role in these studies due to their relationship with quantum groups. Studies related to deformed boson algebras can be extended to deformed fermion algebras [17–22], such that instead of quasi-commutation relations one can consider quasi-anticommutation relations between creation and annihilation operators with appropriate deformation parameters. Since fermions should satisfy the Pauli–Fermi principle, studies related to deformed fermion algebras show extra difficulties as compared with the deformed boson case.

In this paper, we first consider the fermionic system with real and positive deformation parameters p and q. We examine whether this system shows any symmetry. With the extension of this system to the n-dimensional case, we obtain the $SU_{p/q}(n)$ quantum group covariant deformed fermion algebra. We study the $p, q \to 0$ limit which implies the orthofermion algebra. Finally we construct a system with $n(n-1)$ deformation parameters in order to get the most general form. We find that simple commutation relations require some restrictions on the deformation parameters, so that the number of parameters decreases to $2(n-1)$. This system is not invariant under the quantum group unless the number of parameters is reduced to two.

2 Quantum group covariant two parameter deformed fermion algebra

For simplicity, we start our study by considering a 2-dimensional deformed fermion algebra. For this system the annihilation operators can be written

$$c_1 = c \otimes p^N, \tag{2.1}$$
$$c_2 = (-q)^N \otimes c, \tag{2.2}$$

where c is the standard fermion annihilation operator and N is the number operator, satisfying

$$cc^* + c^*c = 1, \tag{2.3}$$
$$N = c^*c, \tag{2.4}$$
$$c^2 = 0. \tag{2.5}$$

Since the deformation parameter depends on the number operator N and $N^2 = N$, any deformation parameter s^N can be written in the form of linear function, namely,

$$s^N = 1 - (1-s)N, \tag{2.6}$$

where s is the complex number and N can take the values 0 and 1.

In a matrix representation c can be taken as

$$c = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}. \tag{2.7}$$

We know that the deformed annihilation operators with their corresponding creation ones, in contrast to a deformed boson system, should obey anticommutation relations in the limit when the deformation parameters approach unity. Due to the Pauli–Fermi principle, the square of both creation and annihilation operators should give zero. If we look at the relations which are obeyed by the operators c_1 and c_2, we see that all expectations are satisfied with the following equations:

$$c_1 c_2 = -\frac{q}{p} c_2 c_1\,, \tag{2.8}$$
$$c_1 c_2^* = -qp c_2^* c_1\,, \tag{2.9}$$

$$c_1^2 = 0, \tag{2.10}$$
$$c_2^2 = 0, \tag{2.11}$$
$$c_1 c_1^* + q^2 c_1^* c_1 = c_2 c_2^* + p^2 c_2^* c_2, \tag{2.12}$$
$$\begin{aligned} c_1^* c_1 + c_2^* c_2 &= c^* c \otimes p^{2N} + q^{2N} \otimes c^* c \\ &= [n_1 + n_2] . \end{aligned} \tag{2.13}$$

Here for the deformed number operator

$$[n] \equiv \frac{p^{2n} - q^{2n}}{p^2 - q^2}, \tag{2.14}$$
$$c_1^* c_1 = n_1 (p^2)^{n_2}, \tag{2.15}$$
$$c_2^* c_2 = n_2 (q^2)^{n_1}. \tag{2.16}$$

n_1 and n_2 are number operators and can only take the values 0 and 1. The $p \to q$ limiting case is discussed in [23]. As seen in the above relations, we are interested in the two parameter deformed fermion algebra generalizing the deformation with only one parameter [22]. If we consider the $p, q \to 1$ limit, then of course, we get the well-known 2-dimensional fermion algebra.

In order to see that this system shows $SU_r(2)$ symmetry with $r = p/q$; it is sufficient to show that our system is invariant under the following transformation:

$$\begin{pmatrix} c_1^{'} \\ c_2^{'} \end{pmatrix} = M \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} a & -rb^* \\ b & a^* \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \tag{2.17}$$

such that M is the transformation matrix and it is an element of $SU_r(2)$. The elements of matrix M satisfy the following equations:

$$ab = rba, \tag{2.18}$$
$$ab^* = rb^* a, \tag{2.19}$$
$$bb^* = b^* b, \tag{2.20}$$
$$aa^* + r^2 b^* b = 1, \tag{2.21}$$
$$a^* a + bb^* = 1. \tag{2.22}$$

If we rewrite all relations satisfied by the deformed annihilation and creation operators (2.8)–(2.13) for the transformed ones, we can easily see that this system remains unchanged. In this computation the matrix elements of M are assumed to commute with c_1, c_1^*, c_2, c_2^* or equivalently in (2.17) when the matrix multiplication is performed the elements of M act on c_1 and c_2 by tensor multiplication.

Before extending our system to the n-dimensional case, studying the 3-dimensional case can be worthwhile. In the 2-dimensional case the construction of the transformation matrix M is clear. For higher-dimensional cases it is more convenient to write M as a product of simpler matrices. In this sense, studying the 3-dimensional case will show us how this system works for higher dimensions in a clear way. Before the construction of the transformation matrix M for the 3-dimensional case, first let us write the annihilation operators for the 3-dimensional deformed fermion algebra:

$$c_1 = c \otimes p^N \otimes p^N, \tag{2.23}$$
$$c_2 = (-q)^N \otimes c \otimes p^N, \tag{2.24}$$
$$c_3 = (-q)^N \otimes (-q)^N \otimes c. \tag{2.25}$$

These operators can satisfy the following relations:

$$c_1 c_2 = -\frac{q}{p} c_2 c_1, \tag{2.26}$$
$$c_1 c_3 = -\frac{q}{p} c_3 c_1, \tag{2.27}$$
$$c_2 c_3 = -\frac{q}{p} c_3 c_2, \tag{2.28}$$
$$c_1 c_2^* = -qp c_2^* c_1, \tag{2.29}$$
$$c_1 c_3^* = -qp c_3^* c_1, \tag{2.30}$$
$$c_2 c_3^* = -qp c_3^* c_2, \tag{2.31}$$
$$c_1^2 = 0, \tag{2.32}$$
$$c_2^2 = 0, \tag{2.33}$$
$$c_3^2 = 0, \tag{2.34}$$
$$c_1 c_1^* + q^2 c_1^* c_1 = c_2 c_2^* + p^2 c_2^* c_2, \tag{2.35}$$
$$c_2 c_2^* + q^2 c_2^* c_2 = c_3 c_3^* + p^2 c_3^* c_3, \tag{2.36}$$
$$c_1^* c_1 + c_2^* c_2 + c_3^* c_3 = [n_1 + n_2 + n_3] . \tag{2.37}$$

In order to see whether this system is quantum group covariant or not, we can consider the following matrix M as the transformation matrix;

$$M = A_{12}(a_1) A_{23}(a_2) A_{12}(a_3) X_{12}(\alpha_1) X_{23}(\alpha_2), \tag{2.38}$$

where

$$A_{12}(a_1) = \begin{pmatrix} a_1 & -\frac{p}{q} a_1^{'} & 0 \\ a_1^{'} & a_1^* & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} (A_1)_{2x2} & \\ & 1 \end{pmatrix}, \tag{2.39}$$

$$A_{23}(a_2) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & a_2 & -\frac{p}{q} a_2^{'} \\ 0 & a_2^{'} & a_2^* \end{pmatrix} = \begin{pmatrix} 1 & \\ & (A_2)_{2x2} \end{pmatrix}, \tag{2.40}$$

$$A_{12}(a_3) = \begin{pmatrix} a_3 & -\frac{p}{q} a_3^{'} & 0 \\ a_3^{'} & a_3^* & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} (A_3)_{2x2} & \\ & 1 \end{pmatrix}, \tag{2.41}$$

$$X_{12}(\alpha_1) = \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{-i\alpha_1} & \\ & & 1 \end{pmatrix}, \tag{2.42}$$

$$X_{23}(\alpha_2) = \begin{pmatrix} 1 & & \\ & e^{i\alpha_2} & \\ & & e^{-i\alpha_2} \end{pmatrix}, \tag{2.43}$$

since any element of $SU_{p/q}(3)$ can be expressed in the form (2.38) [24]. Here A_1, A_2, A_3 are matrices belonging to $SU_{p/q}(2)$ including $(a_1, a_1^{'})$, $(a_2, a_2^{'})$, $(a_3, a_3^{'})$ respectively, where $a_i^{'} = (1 - a_i^* a_i)^{1/2}$. The elements of different matrices commute with each other and they also satisfy the relation

$$a_i a_i^* - \frac{p^2}{q^2} a_i^* a_i = 1 - \frac{p^2}{q^2}, \quad i = 1, 2, 3. \tag{2.44}$$

It can easily be shown that the system is invariant under a transformation by the matrices $X_{12}(\alpha_1)$, $X_{23}(\alpha_2)$, since

α_1, α_2 are phases which are central. Therefore, if we transform the annihilation operators by the transformations

$$\begin{pmatrix} c_1^{'} \\ c_2^{'} \\ c_3^{'} \end{pmatrix} \longrightarrow A_{12}(a_1) \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \tag{2.45}$$

$$\begin{pmatrix} c_1^{'} \\ c_2^{'} \\ c_3^{'} \end{pmatrix} \longrightarrow A_{23}(a_2) \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \tag{2.46}$$

$$\begin{pmatrix} c_1^{'} \\ c_2^{'} \\ c_3^{'} \end{pmatrix} \longrightarrow A_{12}(a_3) \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}, \tag{2.47}$$

and check whether the transformed version of the annihilation operators still satisfy relations (2.26)–(2.37), we get the result that this system is invariant under the $SU_{p/q}(3)$ quantum group.

Now, let us extend our system to n-dimensional case. The annihilation operators can be written

$$c_1 = \underbrace{c \otimes p^N \otimes \ldots \otimes p^N}_{n\text{terms}} \tag{2.48}$$

$$c_2 = \underbrace{(-q)^N \otimes c \otimes p^N \otimes \ldots \otimes p^N}_{n\text{terms}} \tag{2.49}$$

$$\vdots$$

$$c_n = \underbrace{(-q)^N \otimes (-q)^N \otimes \ldots \otimes c}_{n\text{terms}}. \tag{2.50}$$

The system can be described with the following algebra:

$$c_i c_j = -\frac{q}{p} c_j c_i, \quad i \prec j, \quad i = 1, 2, \ldots, n-1,$$
$$j = 2, 3, \ldots, n, \tag{2.51}$$
$$c_i c_j^* = -qp c_j^* c_i, \quad i \neq j,$$
$$i, j = 1, 2, \ldots, n \tag{2.52}$$
$$c_i^2 = 0, \quad i = 1, 2, \ldots, n, \tag{2.53}$$
$$c_i c_i^* + q^2 c_i^* c_i = c_{i+1} c_{i+1}^* + p^2 c_{i+1}^* c_{i+1},$$
$$i = 1, 2, \ldots, n-1, \tag{2.54}$$
$$\sum_{i=1}^{n} c_i^* c_i = [n_1 + n_2 + \ldots + n_n], \tag{2.55}$$

where

$$c_i^* c_i = n_i m_i = n_i (p^2)^{\sum_{k=i+1}^{n} n_k} (q^2)^{\sum_{k=1}^{i-1} n_k}, \tag{2.56}$$
$$c_i c_i^* = (1 - n_i) m_i. \tag{2.57}$$

This system shows $SU_{p/q}(n)$ symmetry and its covariance under the quantum group $SU_{p/q}(n)$ can be seen on consideration of the transformation matrix M (2.38)

$$M = \prod_{k=1}^{n-1} \prod_{i=1}^{n-k} A_{i,i+1}(a_{ik}) \prod_{i=1}^{n-1} X_{i,i+1}(\alpha_i), \tag{2.58}$$

which is an element of $SU_{p/q}(n)$.

Up to now, we were interested with the invariance of the system under special unitary quantum groups. But we can also easily realize that for the 2-dimensional case, by multiplying $X_{23}(\alpha_2)$ with the transformation matrix M (2.17); we get a new matrix $M^{'}$ which is an element of $SU_{p/q}(2) \times U(1) \cong U_q(2)$ and the transformation of the annihilation operators with this consideration still conserve the invariance for the system obeying (2.8)–(2.13). Therefore we can say that this system is covariant under the 2-dimensional unitary quantum group. With the extension of this study to the n-dimensional generalized version, we also get the unitary quantum group covariance for n dimensions.

3 Orthofermion algebra

In this part of our paper, we study the $p, q \to 0$ limit. In the limit $p, q \to 0$, it is not difficult to show that the deformation parameters p^N and $(-q)^N$ can be written as

$$p^N = (-q)^N = cc^* \tag{3.1}$$

considering (2.6). Thus, in view of this relation and (2.48)–(2.50), we can rewrite the annihilation operators as

$$c_1 = \underbrace{c \otimes cc^* \otimes \ldots \otimes cc^*}_{n\text{terms}} \tag{3.2}$$

$$c_2 = \underbrace{cc^* \otimes c \otimes cc^* \otimes \ldots \otimes cc^*}_{n\text{terms}} \tag{3.3}$$

$$\vdots$$

$$c_n = \underbrace{cc^* \otimes cc^* \otimes \ldots \otimes c}_{n\text{terms}}. \tag{3.4}$$

Now, let us write the orthofermion algebra [25] as

$$c_i c_j = 0, \tag{3.5}$$
$$c_i c_j^* = \Pi \delta_{ij}, \quad i, j = 1, 2, \ldots, n, \tag{3.6}$$
$$\Pi^2 = \Pi = \Pi^\dagger, \tag{3.7}$$

where Π is a projection operator [26] and δ_{ij} is the Kroneker delta.

It may easily be verified that the annihilation operators (3.2)–(3.4) satisfy the relations (3.5)–(3.7) with

$$\Pi = \underbrace{cc^* \otimes cc^* \otimes \ldots \otimes cc^*}_{n\text{terms}}. \tag{3.8}$$

Except for the trivial representation the only representations of (3.5)–(3.7) is $(n+1)$-dimensional and coincides with the orthofermion algebra which conventionally is defined by

$$c_i c_j = 0, \tag{3.9}$$
$$c_i c_j^* = \left(1 - \sum_{k=1}^{n} c_k^* c_k\right) \delta_{ij}, \quad i, j = 1, 2, \ldots, n. \tag{3.10}$$

By using the method developed in Sect. 2, it can be shown that these relations are invariant under the action of the

$SU_r(n)$ quantum group for any r. First we choose $n=2$. The transformed creation and annihilation operators of the orthofermion algebra are given by

$$c_1^{'} = a\otimes c_1 - rb^{*}\otimes c_2, \qquad (3.11)$$

$$c_2^{'} = b\otimes c_1 + a^{*}\otimes c_2, \qquad (3.12)$$

where a, a^*, b, b^* satisfy (2.18)–(2.22).

By direct computation, it can be verified that

$$c_i^{'\,2} = 0, \quad i=1,2, \qquad (3.13)$$
$$c_i^{'}c_j^{'} = 0, \quad i\neq j \qquad (3.14)$$

follow from (3.9), whereas

$$c_i^{'}c_j^{'\,*} = 0, \quad i\neq j \qquad (3.15)$$

follows from (3.10) and (2.19). Then

$$c_1^{'\,*}c_1^{'} + c_2^{'\,*}c_2^{'} = c_1^{*}c_1 + c_2^{*}c_2 \qquad (3.16)$$

follows from (2.21) and (2.22), so that

$$c_1^{'}c_1^{'\,*} = c_1c_1^{*}, \qquad (3.17)$$
$$c_1^{'}c_1^{'\,*} = 1 - c_1^{*}c_1 - c_2^{*}c_2, \qquad (3.18)$$
$$c_1^{'}c_1^{'\,*} = 1 - c_1^{'\,*}c_1^{'} - c_2^{'\,*}c_2^{'}. \qquad (3.19)$$

Here the first equality results from using (2.21), the second equality follows from (3.10) and the third equality follows from (3.16). The corresponding equality for $c_2^{'}$, $c_2^{'\,*}$,

$$c_2^{'}c_2^{'\,*} = 1 - c_1^{'\,*}c_1^{'} - c_2^{'\,*}c_2^{'}, \qquad (3.20)$$

is shown in a similar manner by using (2.22) instead of (2.21) in the first step. As a final step, in order to show the $SU_r(n)$ invariance of the orthofermion algebra, we use the method which is developed in Sect. 2.

Note that (3.5)–(3.7) defines a 2^n-dimensional representation and is therefore reducible. However the spectrum of the generalized number operator,

$$[N] = \frac{p^{2N}-q^{2N}}{p^2-q^2} = 0, 1, p^2+q^2, \ldots, \qquad (3.21)$$

in the limit $p, q \to 0$ becomes the correct orthofermion particle number operator.

In order to see what this means in a clear way, let us consider the 2-dimensional case as an example. The annihilation operators are

$$c_1 = c\otimes cc^{*}, \qquad (3.22)$$
$$c_2 = cc^{*}\otimes c, \qquad (3.23)$$

and they satisfy the following relations:

$$c_1^2 = 0, \qquad (3.24)$$
$$c_2^2 = 0, \qquad (3.25)$$
$$c_1c_2 = 0, \qquad (3.26)$$
$$c_1c_2^{*} = 0, \qquad (3.27)$$
$$c_1c_1^{*} = c_2c_2^{*} = \Pi = 1 - (c_1^{*}c_1 + c_2^{*}c_2 + c^{*}c\otimes c^{*}c). \qquad (3.28)$$

At first glance, one may think that there is some problem in (3.28). This is because when we compare this equation with (3.10) for $i, j = 1, 2$, we have $c^*c\otimes c^*c$ as an extra term. But it can be realized that the above consideration includes the reducible representation and this term comes from the trivial representation such that it does not contribute other than zero.

Notice that with the consideration of the annihilation operators in the tensor product notation, we get the 4-dimensional representations such that

$$c_1 = \begin{pmatrix} 0 & 1 & 0 & \vdots & 0 \\ 0 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 0 & \vdots & 0 \\ \cdots & \cdots & \cdots & \vdots & \cdots \\ 0 & 0 & 0 & \vdots & 0 \end{pmatrix}, \qquad (3.29)$$

$$c_2 = \begin{pmatrix} 0 & 0 & 1 & \vdots & 0 \\ 0 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 0 & \vdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \vdots & 0 \end{pmatrix}. \qquad (3.30)$$

By considering this block form, we can say that this representation is reducible and its nontrivial irreducible form can be written

$$c_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad (3.31)$$

$$c_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad (3.32)$$

as stated in [26]. With this irreducible form we get the relations which are exactly the same as (3.9) and (3.10).

4 Multiparameter deformed fermion algebra

Up to here, we considered only p and q as deformation parameters. One question is what happens if we deform our system with more than two parameters. To answer this, first let us try to write our annihilation operators in their general form,

$$c_1 = c\otimes f_{12}(N)\otimes f_{13}(N)\otimes\ldots\otimes f_{1d}(N), \qquad (4.1)$$
$$c_2 = g_{21}(N)\otimes c\otimes f_{23}(N)\otimes\ldots\otimes f_{2d}(N), \qquad (4.2)$$
$$c_3 = g_{31}(N)\otimes g_{32}(N)\otimes c\otimes f_{34}(N)\otimes\ldots\otimes f_{3d}(N), \qquad (4.3)$$
$$\vdots$$
$$c_d = g_{d1}(N)\otimes\ldots\otimes g_{d(d-1)}(N)\otimes c, \qquad (4.4)$$

where all the functions f and g are of the most general form and $d = 1, 2, \ldots, n$. Since $N^2 = N$ and the deformation parameters depend on the number operator N, these functions can be written in the form of a linear function without losing their generalization. Therefore we can write the deformation parameter $f_{ij}(N)$:

$$f_{ij}(N) = a_{ij} + b_{ij} N = a_{ij}(1 + (p_{ij} - 1)N) = a_{ij} p_{ij}^N. \quad (4.5)$$

The normalization constants a_{ij} can be factored from the direct product and absorbed into the definition of annihilation operators. With (4.5) the annihilation operators can be rewritten as

$$c_1 = c \otimes p_{12}^N \otimes p_{13}^N \otimes \ldots \otimes p_{1d}^N, \quad (4.6)$$

$$c_2 = (-q_{21})^N \otimes c \otimes p_{23}^N \otimes p_{24}^N \otimes \ldots \otimes p_{2d}^N, \quad (4.7)$$

$$c_3 = (-q_{31})^N \otimes (-q_{32})^N \otimes c \otimes p_{34}^N \otimes \ldots \otimes p_{3d}^N, \quad (4.8)$$

$$\vdots$$

$$c_d = (-q_{d1})^N \otimes \ldots \otimes (-q_{d(d-1)})^N \otimes c. \quad (4.9)$$

If we study the relations satisfied by these annihilation operators, we can easily see that in order to get simple commutation relations, as in (2.54), we should consider some restrictions which are related with the deformation parameters. Our restriction for the deformation parameters p_{ij} is that all p_{ij} should be equal among themselves for all fixed j-values, where $i \prec j$, namely,

$$p_{13} = p_{23},$$
$$p_{14} = p_{24} = p_{34},$$
$$\vdots$$
$$p_{1(n-1)} = p_{2(n-1)} = \ldots = p_{(n-2)(n-1)}.$$

On the other hand, the restriction for the deformation parameters q_{ji} is that all q_{ji} should be equal among themselves for all fixed i-values for all $i \prec j$, that is to say

$$q_{21} = q_{31} = \ldots = q_{n1},$$
$$q_{32} = q_{42} = \ldots = q_{n2},$$
$$\vdots$$
$$q_{(n-1)(n-2)} = q_{n(n-2)}.$$

With these considerations the number of deformation parameters decreases from $d(d-1)$ to $2(n-1)$. Thus, our system can be written in the most general form as

$$c_i c_j = -\frac{q_{ji}}{p_{ij}} c_j c_i, \quad i \prec j,$$
$$i = 1, 2, \ldots, d-1, \quad j = 2, 3, \ldots, d, \quad (4.10)$$

$$c_i c_j^* = -q_{ji} p_{ij} c_j^* c_i, \quad i \neq j,$$
$$i, j = 1, 2, \ldots, d, \quad (4.11)$$

$$c_i^2 = 0, \quad i = 1, 2, \ldots, d, \quad (4.12)$$

$$c_i c_i^* + q_{ji}^2 c_i^* c_i = c_j c_j^* + p_{ij}^2 c_j^* c_j,$$
$$i = 1, 2, \ldots, d-1, \quad j = i+1, \quad (4.13)$$

$$\sum c_i^* c_i = [n_1 + n_2 + \ldots + n_n]. \quad (4.14)$$

This system is not invariant under the q-deformed unitary quantum group.

5 Conclusion

In this paper, we studied a deformed system of n fermions with two deformation parameters p and q. We have shown that it possesses $SU_{p/q}(n)$ symmetry. When the two deformation parameters are equal to each other, we obtain the multi-dimensional fermionic Newton oscillator [23] which possesses $SU(n)$ symmetry. In the $p, q \to 1$ limit, we get the well-known system for n ordinary fermions, whereas for $p, q \to 0$ we obtain the 2^n-dimensional (reducible) representation of the orthofermion algebra. We studied the $n = 2$ case in detail and showed that this representation is the combination of the trivial and irreducible representation. A consideration of the irreducible representation will give us the orthofermion algebra in the usual sense. Another remarkable point is that this system posssesses $SU_r(n)$ symmetry for any r since the $p, q \to 0$ limit can be chosen to obtain any $r = p/q$. Finally, we constructed a system which generalizes the n-dimensional two parameter deformed fermion algebra by increasing the number of deformation parameters to $2n - 2$.

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